

Experimental Verification of Modified Command Shaping Using a Flexible Manipulator

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ABSTRACT

A brief discussion introduces the original input shaping method applied to a system with varying parameters. A change in parameters causes a vibration in the system and a modified command shaping technique is created to eliminate this unwanted motion. Using a two degree of freedom (DOF), flexible manipulator, experiments are conducted by varying perturbations to input trajectories to compare the modified method to a previously used adaptive proportional plus derivative (P.D.) control method. The control scheme that produces the smaller magnitude of resonant vibration at the first natural frequency of the robot is considered the more effective control method.

KEY WORDS: command shaping, flexible manipulator, adaptive P.D. control, vibration reduction

INTRODUCTION

With an increase in environmental concern, the interest in long-reach, flexible manipulators has grown significantly in the last few years. The long, slender links of the flexible manipulator have a desirable strength-to-weight ratio and are cheaper to build than their rigid link counterparts. However, the inherent flexibility of the links generates a residual vibration that makes end-point positioning of the tip difficult. In most robotic applications, accurate end-point positioning is desired. Therefore, a method to eliminate the residual vibration of a flexible manipulator is the main focus of this paper.

The reduction of residual vibration in a manipulator is not a new idea. Many active and passive control schemes exist that were designed to eliminate vibration. One of the most rudimentary passive approaches is to move the robot to a desired location and wait for the vibration to subside. For example, the Space Shuttle Remote Manipulator system is very inefficient in task completion time. Other passive methods include the addition of a second mass which behaves like a vibration absorber or the application of a visco-elastic material that absorbs vibrational kinetic energy [1].

The active control strategies include spectral methods where the command signal is modified so that vibration is not induced into the system. Meckl and Seering [2] developed a direct relationship between the frequency spectrum of the input signal and the resulting vibration. By reducing the spectral magnitude of the input at the resonances of the system, the residual vibration is eliminated. Their work showed promising results if the change in system resonant frequency was less than about 10%. Later, they extended their work to develop a set of force profiles that will accelerate a system to a given velocity level with minimum residual vibration [3].

Another way of reducing the spectral magnitude at the resonances is to add zeros to the system function at the locations of the poles. Singer and Seering [4,5] developed an input shaping technique that alters commanded inputs using the characteristics of the system. Each sample of the input is transformed into a new set of impulses that do not excite the system resonances. However, this method cannot accommodate systems that contain time varying parameters so a modified shaping method is developed. To fully appreciate the modified command shaping technique, Singer and Seering's input shaping method will now be presented.

INPUT SHAPING METHOD

The dynamic equations of a manipulator become quite complex when the flexibility of the links is considered. To greatly simplify the problem, the dynamics of the manipulator are assumed to be a linear combination of flexible and rigid body motion. The flexible motion is often called the residual vibration of the system where as the desired trajectory is the rigid body motion. The original input shaping method developed by Singer and Seering models the residual vibration of a system as a simple, second-order system. The impulse response of a linear, time-invariant, underdamped second-order system can be written as

$$x(t) = \frac{A \omega_n e^{-\zeta \omega_n (t-t_0)}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2} (t-t_0)) \quad (1)$$

where A is the amplitude of the impulse, ω_n is the natural frequency of the system, ζ is the damping ratio, t is the time and t_0 is the time when the impulse occurs. Since the input shaping technique generates a new set of impulses, the method will be explained using a set of two impulses. The second-order system response to a set of two input impulses is

$$x(t) = B_1 \sin(\alpha t + \phi_1) + B_2 \sin(\alpha t + \phi_2) \quad (2)$$

where

$$B_k = \frac{A_k \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n (t-t_{0k})} \quad (3)$$

$$\alpha = \omega_n \sqrt{1-\zeta^2} \quad (4)$$

$$\phi_k = -\omega_n t_{0k} \sqrt{1-\zeta^2} \quad (5)$$

and t_{0k} signifies the time at which the k^{th} impulse occurs. The two impulse input response given in Equation (2) can be simplified to yield

$$x(t) = B_{amp} \sin(\alpha t + \psi) \quad (6)$$

where

$$B_{amp} = \sqrt{[B_1 \cos(\phi_1) + B_2 \cos(\phi_2)]^2 + [B_1 \sin(\phi_1) + B_2 \sin(\phi_2)]^2} \quad (7) \quad \psi = \tan^{-1} \left(\frac{B_1 \sin(\phi_1) + B_2 \sin(\phi_2)}{B_1 \cos(\phi_1) + B_2 \cos(\phi_2)} \right) \quad (8)$$

Since the purpose of the input shaping method is to eliminate vibration, the amplitude of vibration, Equation (7), must equal zero after the second impulse occurs. This only occurs if the squared terms are independently zero since the sine and cosine functions are orthogonal. The resulting set of equations are

$$B_1 \cos(\phi_1) + B_2 \cos(\phi_2) = 0 \quad (9)$$

$$B_1 \sin(\phi_1) + B_2 \sin(\phi_2) = 0 \quad (10)$$

which can be solved for the two impulse amplitudes and the times at which they occur. Since Equations (9) and (10) are transcendental, an infinite number of solutions exist. Therefore, only the solution that yields the shortest time duration between impulses with positive amplitudes is chosen. The resulting solution is

$$A_1 = \frac{1}{1+M} \quad (11)$$

$$t_{01} = 0 \quad (12)$$

$$A_2 = \frac{M}{1+M} \quad (13)$$

$$t_{02} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} \quad (14)$$

$$M = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}} \quad (15)$$

From Equation (14), notice that the second impulse occurs at one-half the damped oscillation period of the system. Intuitively, the second impulse creates a vibration that attempts to cancel the vibration generated by the first impulse. The amount of remaining residual vibration is defined by a vibration error expression. A detailed discussion of the vibration error can be found in previous work by Magee [6].

TIME VARYING PARAMETERS

Although the original input shaping technique eliminates residual vibration, the method does not address the issue of varying system parameters. For many flexible robot systems, the natural frequency and damping ratio are functions of position. As the robot moves through joint space the parameters change. The effect of time varying parameters on the input shaping method will now be discussed and then a modified command shaping technique

is presented.

To develop this new technique, the implementation of the original input shaping algorithm to a discrete-time system is presented. In the general case, for each sample of the input, the input shaping method creates a set of N impulses. From Equation (14), the output impulses are equally spaced in time with a continuous-time period, denoted $delT$, of

$$delT = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \quad (16)$$

which is a function of both the natural frequency, ω_n , and damping ratio, ζ . To utilize this time period information in a discrete-time control system, the continuous-time data must be represented in discrete-time. From discrete-time signal processing, a continuous-time signal, $x(t)$, is represented mathematically as a sequence of numbers, $x[n]$, where n is strictly an integer. To transform the continuous-time period $delT$ into a discrete-time period, $deln$, the sampling rate of the discrete-time system, f_s , is used. The equation to perform this transformation is

$$deln = \text{int}(delT * f_s) \quad (17)$$

where the int function truncates its argument to an integer.

For Singer's original input shaping method, the discrete-time period, $deln$, never changes because the system parameters are assumed constant. When the input shaping method is applied to a system with time varying parameters, the continuous-time period, $delT$, becomes time varying as well. A significant change in $delT$ will result in a change in the discrete-time period, $deln$, which produces a vibration in the system. This induced vibration will be verified later in this paper.

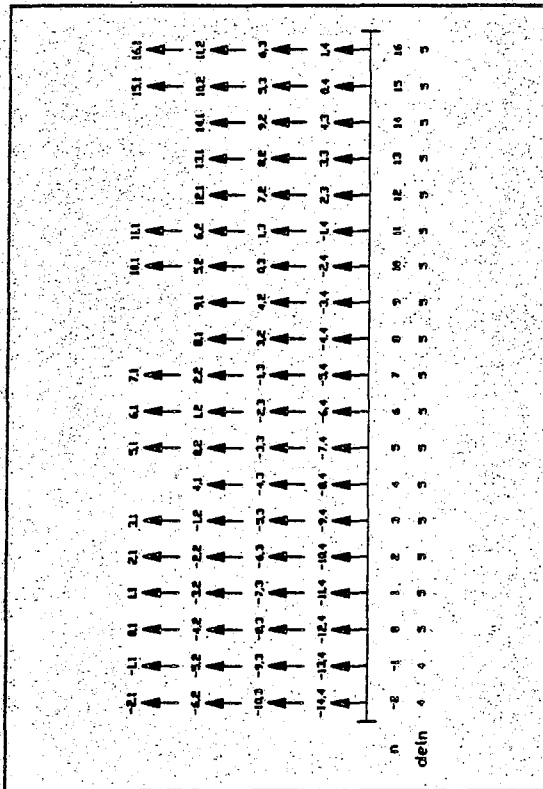


Fig.1 Steady-State Output for an Increase in Discrete-Time Period Using Input Shaping

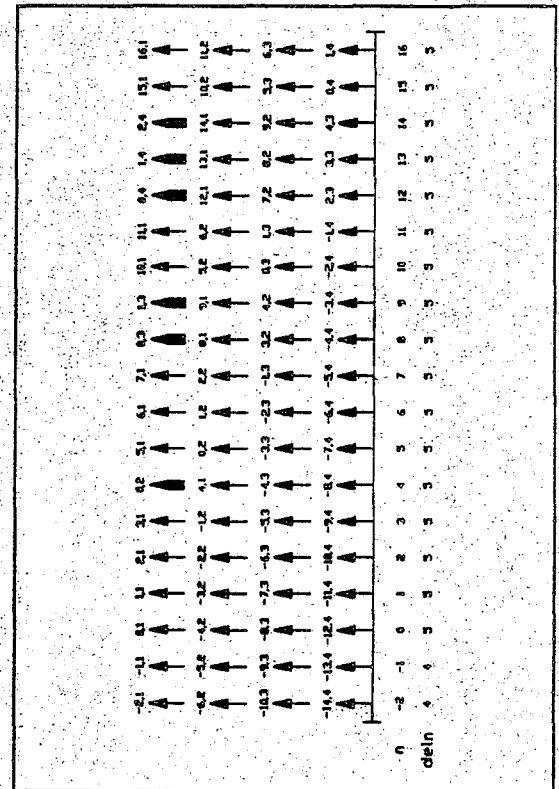


Fig.2 Steady-State Output for an Increase in Discrete-Time Period Using MCS

The effect of time varying parameters on the input shaping method can also be demonstrated graphically. Consider the input shaping method that produces four new impulses for each sample of the desired input. Now assume that a change in system configuration alters the value of the discrete-time period. Figure 1 shows the steady-state impulse output for a change in discrete-time period, $deln$, from four to five. Each output impulse is designated $\{a,b\}$ where a indicates the discrete-time location of the input sample responsible for the four output impulses and b indexes the four resulting output impulses. After examining Fig.1, it is obvious that the change in

$deln$ has caused gaps in the output for discrete values of n . At $n=4$, for example, only three impulses contribute to the overall output. To make matters worse, this lack of impulses is repeated five more times at a discrete-time period near the system's natural period. This phenomenon induces a vibration into the system that is caused solely by the application of the input shaping method to a system with time varying parameters. The vibration is also present when the value of $deln$ decreases. In this case, an excess of impulses is created which are spaced at integer multiples of the system's natural period [7].

To eliminate the induced vibration, a modified command shaping method is proposed to make the impulse output more uniform when a change in $deln$ is encountered. To compensate for the change in discrete-time period, extra impulses are added for an increase in $deln$ and impulses are removed for a decrease in $deln$. The choice of which impulses are affected is based on the number of output impulses from the shaping algorithm and the old and new values of the discrete-time period.

MODIFIED COMMAND SHAPING

The previous section showed that Singer's original input shaping method produces gaps in the output for systems with time varying parameters. To smooth the output, a modified command shaping method is developed. When the discrete-time period increases, designate the discrete-time value as $n=0$. The modified command shaping technique transforms the next $N-1$ samples of the input, i.e. $0 \leq n \leq N-2$, using both the old and new values of $deln$. Using the new value of $deln$, the input sample is shaped to create N output impulses that are added to the steady-state output at their respective discrete-time values. Using the old value of $deln$, the same input sample is also shaped to create N output impulses. However, only the last $N-(n+1)$ output impulses are added to the steady-state output at their respective discrete-time values. For discrete-time values of $n \geq N-1$, each sample of the input is shaped normally using the new value of $deln$ to generate the N output impulses.

The steady-state impulse output for an increase in discrete-time period using the modified shaping method is shown in Fig.2. The darkened impulses emphasize the impulses added by the modified shaping method. Comparing this output to Fig.1, it is apparent that the new impulse profile is much smoother than the original input shaping method.

The modified command shaping method also works for a decrease in the discrete-time period. Designating the discrete-time value as $n=0$, the input sample is shaped only once using the new value of $deln$ to produce N output impulses. Instead of adding all of the output impulses, only the first $(n+1)$ output impulses are added to the steady-state output at their respective discrete-time values. By manipulating the steady-state output in this manner, the extra impulses that are added for the increase in discrete-time period are the same indexed impulses that are removed when $deln$ decreases.

COMPARISON OF THE SHAPING METHODS

By shaping the error term in an adaptive feedback control system developed by Yuan [8], the two shaping methods are now compared using a precomputed desired trajectory. The principal trajectory consists of a circle in cartesian space that is three feet in diameter with a period of nine seconds. The circle was positioned in the workspace of the manipulator so that a change in the discrete-time period would occur. To measure the vibration of the system, an accelerometer was mounted at the tip of the manipulator. A signal analyzer records the time response of the accelerometer and computes the frequency response of the data. To ensure reliable averaging of the data, the manipulator is commanded to follow the desired trajectory eight times.

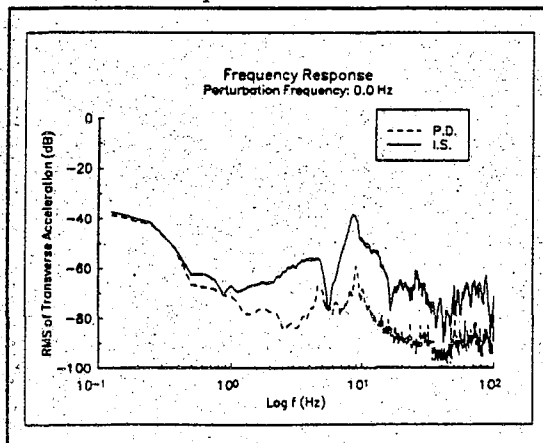


Fig.3 Frequency Response of RALF
P.D. vs. Input Shaping

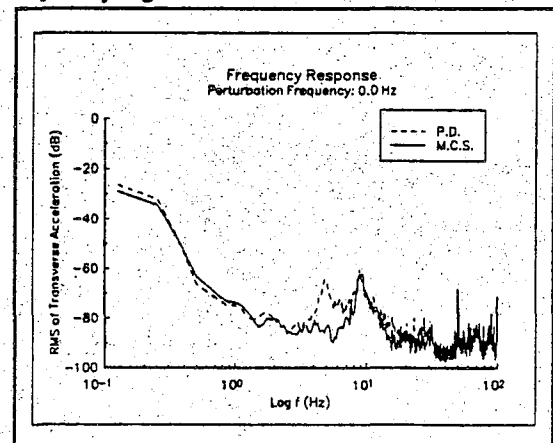


Fig.4 Frequency Response of RALF
P.D. vs. Modified Command Shaping

The first comparison is between Singer's input shaping method and Yuan's adaptive P.D. control structure. Figure 3 presents the spectral comparison for these two techniques. Notice that the magnitude of the frequency response for the input shaping method is much greater than for the adaptive P.D. routine alone. At the frequency range that the input shaping method is designed to control (3.7-5.5 Hz), the frequency response is 12 dB larger. This is due to a change in discrete-time period which induces vibration at the system resonance. It is also worth noting that the second resonance of RALF occurs near 10 Hz and is also very evident in Fig.3.

The next comparison is between the modified command shaping method and Yuan's adaptive P.D. control algorithm. The spectral comparison for these two methods is shown in Fig.4. From this figure, the modified method has reduced the magnitude of the resonant vibration at the first frequency range by almost 20 dB. Therefore, the modified method has reduced the vibration by nearly 32 dB over the input shaping method. Again, this increase in performance is due to the inability of the input shaping method to accommodate a change in discrete-time period.

DESIRED TRAJECTORIES WITH PERTURBATIONS

Now that the modified shaping method controls systems with varying parameters, the circle trajectory used previously will be altered. A sinusoidal perturbation with variable frequency is added to the radius parameter of the circle to simulate the frequency components of possible telerobotic inputs. The frequency response of the modified method will be compared to that of the adaptive P.D. routine to determine the better control scheme.

The first trajectory, shown in Fig.5, contains a 1 Hz sine wave with an amplitude of 1.5" riding on the radial component of the circle. Notice that this trajectory has nine "bumps" around the circle since the period is nine seconds. The frequency response comparison for this trajectory is shown in Fig.6. Since the command shaping technique was not designed to eliminate 1 Hz vibration, the two control schemes show comparable results for this frequency range. However, the modified method reduces the magnitude of vibration by 18 dB at the first natural frequency value of 4.8 Hz. This results in a vibration that is 1.6% of the amplitude of the original adaptive P.D. vibration at this particular frequency.

The second trajectory contains a 4.8 Hz sine wave added to the principal circle and is designed to excite the first natural frequency of RALF. Figure 7 displays the frequency response comparison for this trajectory. The difference in magnitude is 32 dB at the first natural frequency which corresponds to 0.06% of the adaptive P.D. vibration at this frequency value.

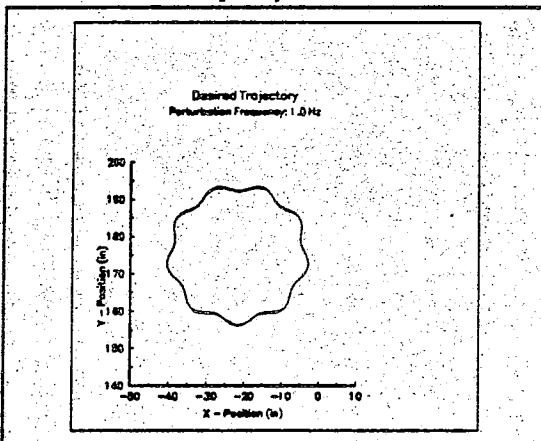


Fig.5 Circle Trajectory with 1 Hz Perturbation

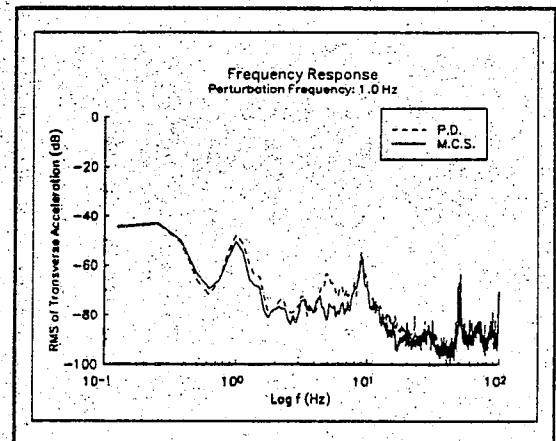


Fig.6 Frequency Response of RALF
P.D. vs. Modified Command Shaping
1 Hz Perturbation

The last circular trajectory was designed to have a perturbation resonance above the first natural frequency of RALF. For this trajectory, a 10 Hz sine wave is added to the principal circle. Figure 8 shows a reduction in magnitude of the frequency response at the 4.8 Hz frequency location of 9 dB. This results in a vibration that is 12.6% of the amplitude of the original adaptive P.D. vibration for this particular frequency. For all three test cases, the modified shaping method was able to reduce vibration at the first natural frequency of the manipulator.

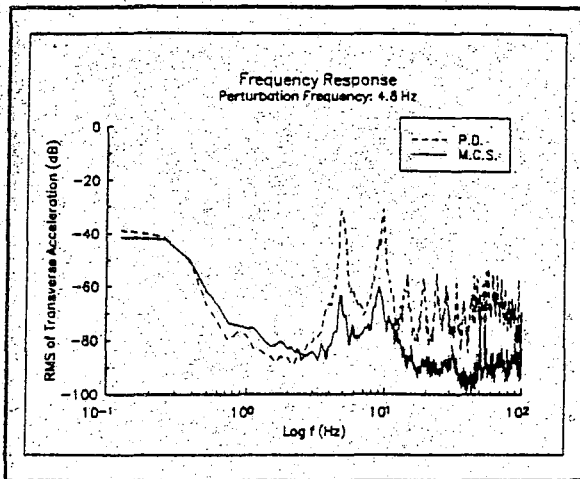


Fig.7 Frequency Response of RALF
P.D. vs. Modified Command Shaping
4.8 Hz Perturbation

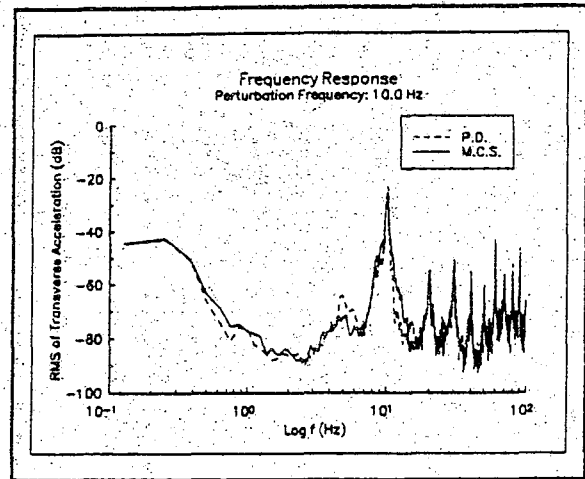


Fig.8 Frequency Response of RALF
P.D. vs. Modified Command Shaping
10 Hz Perturbation

CONCLUSION

The original input shaping method developed by Singer and Seering was presented and was shown to induce vibration in a manipulator that contains time varying parameters. To eliminate the induced vibration, a modified command shaping technique was developed. The modified method was able to reduce vibration at the first natural frequency of the system for input trajectories containing different sinusoidal perturbations. Thus, the modified shaping method offers considerable vibration reduction for inputs with multiple frequency components.

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